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RADIATIVE ENERGY TRANSFER FROM A SMALL SPHERE†

GEORGE EMANUEL

Aerodynamics and Propulsion Research Laboratory, Aerospace Corporation, El Segundo, California

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NOMENCLATURE

a, radius of sphere;
B_u, Bouguer number, $a\alpha$;
E_n, exponential integral of order n;
I, intensity;
q, Hopf's function;
$\bar{q}$, radial heat flux;
Q, dimensionless heat flux, $r^2\bar{q}/\sigma(T_w^4 - T_\infty^4)a^2$;
Q_i, terms in the expansion of Q;
r, radius measured from the center of the sphere;
T, temperature;
z, outer coordinate, αr.

Greek symbols
α, constant volumetric absorption coefficient;
$\Delta_i, \delta_i, \nu_i$, coefficients in the asymptotic expansions;
$\theta_i^{(i)}$, terms in the inner expansion of $\tilde{\Theta}$;
$\theta_i^{(o)}$, terms in the outer expansion of $\tilde{\Theta}$;
Θ, dimensionless emissive power, $(T^4 - T_\infty^4)/(T_w^4 - T_\infty^4)$;
$\tilde{\Theta}$, modified emissive power, $z\Theta(z)$;
μ, $\cos\phi$;
μ_m, $\cos\phi_m = (r^2 - a^2)^{\frac{1}{2}}/r$;
ρ, inner coordinate, $(r - a)/a$;
σ, Stefan–Boltzmann constant;
ϕ, angle between radius vector and intensity.

Subscripts
a, denotes conditions in the gas at the surface of the sphere;
w, denotes surface condition;
∞, denotes condition at infinity.

Superscripts
(c), denotes composite quantity;
(i), denotes inner-region quantity;
(0), denotes outer-region quantity.

1. INTRODUCTION

RADIATIVE transfer in a spherically symmetric medium has recently received considerable emphasis. Radiative transfer between concentric spheres with a heat-generating gas has been investigated in [1–4]. The concentric sphere problem with an emitting-absorbing grey gas has been studied in [3–8]. The emphasis in these papers, as in this one, is the determination of the emissive power and radiant heat flux. This is in contrast to the astrophysical approach, in which the specific intensity is of prime interest.

This work investigates the radiative transfer from a single sphere of radius a situated in a quiescent nonconducting gas with a constant volumetric absorption coefficient α. Although this is a limiting concentric sphere case with the outer sphere at infinity, none of the aforementioned references, except [6], considered this limit. Since [6] gives the asymptotic solution for large Bouguer number, $B_u = a\alpha$, we concentrate on the small B_u case. As in [6], the method of matched asymptotic expansions is used. When B_u is small, the gas close to the sphere is optically thin, but away from the sphere, because of its infinite extent, it is optically thick. The asymptotic approach has the important advantages of mathematical simplicity and physical clarity. In particular, the leading terms in the inner and outer expansions for the emissive power represent, in themselves, simple physical situations.

Related work in neutron transport theory is contained in [9] and [10], which give an asymptotic diffusion solution for large distance from the sphere. This solution is only part of the outer expansion and, by itself, would not match the inner expansion.

2. ANALYSIS

This work is limited to radiative energy transfer, consequently the divergence of the heat flux is zero. Let $\bar{q}$ be the net radial heat flux, then $r^2\bar{q}$ is constant. A dimensionless heat flux Q, proportional to $r^2\bar{q}$ is used in the analysis. It is so defined that $Q = 1$ when the gas is transparent, otherwise $|Q| < 1$. Our definitions of Θ and Q are the same as those in in [6], hence the equation for the heat flux is [6]

$$2B_u^2 Q = -(1 + z - B_u)\exp[-(z - B_u)] + (z + B_u)^2 \times E_3(z - B_u) + [1 + (z^2 - B_u^2)^{\frac{1}{2}}]\exp.[-(z^2 - B_u^2)^{\frac{1}{2}}] - (z^2 - B_u^2)E_3[(z^2 - B_u^2)^{\frac{1}{2}}]\cdot$$

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$$+ 2\int_{B_u}^{z} \tilde{\Theta}(z_1)\{\exp[-(z-z_1)] + (z+z_1)E_2(z-z_1)\}\,dz_1$$

$$+ 2\int_{z}^{\infty} \tilde{\Theta}(z_1)\{\exp[-(z_1-z)] - (z+z_1)E_2(z_1-z)\}\,dz_1$$

$$- 2\int_{B_u}^{\infty} \tilde{\Theta}(z_1)\{\exp[-(z^2-B_u^2)^{\frac{1}{2}} - (z_1^2-B_u^2)^{\frac{1}{2}}]$$

$$+ [(z^2-B_u^2)^{\frac{1}{2}} - (z_1^2-B_u^2)^{\frac{1}{2}}]\,E_2[(z^2-B_u^2)^{\frac{1}{2}} + (z_1^2-B_u^2)^{\frac{1}{2}}]\}\,dz_1. \quad (1)$$

The independent variable is the optical depth $z = \alpha r$, which also serves as the outer-region independent variable, and, for convenience, we use a modified emissive power $\tilde{\Theta} = z\Theta(z)$. An integral equation for $\tilde{\Theta}$ is obtained by differentiating equation (1) with respect to z

$$2\tilde{\Theta}(z) = E_3[(z^2-B_u^2)^{\frac{1}{2}}] - E_3(z-B_u) + B_u E_2(z-B_u)$$

$$+ \int_{B_u}^{\infty} \tilde{\Theta}(z_1)\{E_1(|z_1-z|) - E_1[(z^2-B_u^2)^{\frac{1}{2}} + (z_1^2-B_u^2)^{\frac{1}{2}}]\}\,dz_1. \quad (2)$$

Equations (1) and (2) are valid for all B_u; they constitute the basic equations for the subsequent analysis. Equation (2), which was first given by Marshak [9], can be derived by alternative methods. For example, since the divergence of the heat flux is zero, the emissive power is given by

$$4\sigma T^4 = \int_0^{4\pi} I\,d\Omega = 2\pi \int_{-1}^{1} I\,d\mu. \quad (3)$$

We then obtain equation (2) when equation (3) is combined with the solution of the transfer equation for the intensity I.

Outer expansion

We insert the following outer expansion

$$\tilde{\Theta}^{(o)}(z; B_u) = \delta_0(B_u)\,\theta_0^{(o)}(z) + \delta_1(B_u)\,\theta_1^{(o)}(z) + \ldots \quad (4),$$

valid for small B_u where $\delta_0(B_u) \gg \delta_1(B_u) \gg \ldots$, into equation (2). In addition, the exponential integral functions are expanded for small B_u and by means of their recurrence relations, we obtain for the first approximation

$$\theta_0^{(o)}(z) = \tfrac{1}{4}E_0(z) + \tfrac{1}{2}\int_0^{\infty} \theta_0^{(o)}(z_1)[E_1(|z_1-z|) - E_1(z+z_1)]\,dz_1, \quad (5)$$

where $\delta_0 = B_u^2$ and $E_0(z) = (e^{-z}/z)$. It should be noted that both $E_0(0)$ and $E_1(0)$ are infinite and thus equation (5) is a singular integral equation. As we demonstrate later, $\theta_0^{(o)}(0)$ is also infinite.

We also use an expansion for the heat flux

$$Q(B_u) = \nu_0(B_u)Q_0 + \nu_1(B_u)Q_1 + \ldots, \quad (6)$$

where the Q_i are constants. Since Q does not depend on r, expansion (6) is valid in both the outer and inner regions and therefore is uniformly valid to begin with. By means of expansions (4) and (6), we obtain from equation (1)

$$Q_0 = e^{-z} + 2\int_0^{z} \theta_0^{(o)}(z_1)\,[E_3(z-z_1) + zE_2(z-z_1)]\,dz_1$$

$$+ 2\int_z^{\infty} \theta_0^{(o)}(z_1)\,[E_3(z_1-z) - zE_2(z_1-z)]\,dz_1$$

$$- 2\int_0^{\infty} \theta_0^{(o)}(z_1)\,[E_3(z+z_1) + zE_2(z+z_1)]\,dz_1, \quad (7)$$

where $\nu_0(B_u) = 1$. Note that we can derive equation (5) by differentiating equation (7). We determine the constant Q_0 by setting $z = 0$. This yields $Q_0 = 1$, and hence the uniformly valid first approximation for the heat flux is its transparent value. The second approximation is determined in the inner-region subsection. This result for Q_0 and the singular nature of $\theta_0^{(o)}(z)$ have a simple physical interpretation, which we now discuss.

The energy emitted per second by the surface of a black sphere is $4\pi a^2 \bar{q}_w = 4\pi a^2 \sigma T_w^4$. For small B_u we can imagine, with equal regard, either α small with $a = 1$ or the converse. In the first interpretation, with $a = 1$, we obtain the transparent limit, i.e. $Q = 1$, as $B_u = \alpha \to 0$. In the second interpretation, when $B_u = a \to 0$, the sphere shrinks to a point, and if T_w is constant, the emitted flux goes to zero. In this situation, the solution of equations (1) and (2) is the trivial one of $Q = 0$ and $\Theta(z) = 0$ with $z \neq 0$. This solution applies when $T_w \leqslant T_\infty$, and the sphere is a point sink. Since the point-sink solution is of less interest than the point-source one, we assume $T_w > T_\infty$ (in the second interpretation) and require the sphere to emit a constant flux as it shrinks to a point source. Hence, $a^2 T_w^4$ is constant and $T_w \to \infty$ as $a \to 0$. The temperature of the gas adjacent to the sphere, T_a, also goes to infinity as $a \to 0$. From the point of view of the outer region, to a first approximation the sphere looks like a point source with an infinite temperature. Our second interpretation thus explains why $\theta_0^{(o)}(0)$ is infinite. This infinity is eliminated later when we form a composite emissive power $\Theta^{(c)}$. The $Q = 1 + \ldots$ result is also consistent with this interpretation, since a point source can emit, but not absorb, photons.

Equations (5) and (7) (with $Q_0 = 1$) are equivalent equations for a point source in an infinite medium. We obtain their exact solution from neutron transport theory. From equation (3), we note that the average intensity $\int I\,d\Omega$ equals $4\sigma T^4$. The point-source neutron solution in [11] is for the average intensity minus its value at infinity. In our notation, this solution is [12]

$$\theta_0^{(o)} = \frac{3}{4} + \frac{1}{4}\int_1^{\infty} \left\{1 + \frac{1}{s}\ln\left(\frac{s-1}{s+1}\right) + \frac{1}{4s^2} \times \left[\pi^2 + \ln^2\left(\frac{s-1}{s+1}\right)\right]\right\}^{-1} e^{-zs}\,ds. \quad (8)$$

The quantity $[4\theta_0^{(o)}(z)/3]^{-1}$ is tabulated in [13]. We can also accurately approximate $\theta_0^{(o)}$, for all values of z [12], by

$$\theta_0^{(o)}(z) = \frac{1}{4}E_0(z) + \frac{3}{4} - \left(\frac{\pi^2 - 8}{16}\right)E_2(z). \quad (9)$$

In the rest of this work, because of its simplicity, equation (9) is used instead of equation (8). The first term on the right-hand side of equation (9) contains the singularity that dominates when z is small. The second term gives the behaviour for large z. It can be obtained directly by means of the Rosseland diffusion approximation [10, 12].

Inner expansion

In the inner region adjacent to the sphere, we use as the independent variable of order unity a coordinate, $\rho = (r - a)/a$, based on the sphere's radius. Hence, we have $z = B_u(1 + \rho)$ and

$$\tilde{\Theta}(z) = z\Theta(z) = B_u(1 + \rho)\Theta(\rho) = B_u\tilde{\Theta}(\rho), \quad (10)$$

where $\tilde{\Theta}(\rho) = (1 + \rho)\Theta(\rho)$. The inner expansion is written as

$$\tilde{\Theta}^{(i)}(\rho; B_u) = \Delta_0(B_u)\theta_0^{(i)}(\rho) + \Delta_1(B_u)\theta_1^{(i)}(\rho) + \ldots, \quad (11)$$

where $\Delta_0 \gg \Delta_1 \gg \ldots$, and again we limit the analysis to the first approximation. By substituting the foregoing into equation (2) and then expanding for small B_u, we obtain $\Delta_0(B_u) = 1$ and

$$\theta_0^{(i)}(\rho) = \tfrac{1}{2}[\rho + 1 - (2\rho + \rho^2)^{\frac{1}{2}}]. \quad (12)$$

The integral in equation (2) does not contribute to the first approximation; only the inhomogeneous terms contribute to equation (12).

As in the outer approximation, the first inner term for Θ, given by $B_u\theta_0^{(i)}(\rho)/z$, has a simple physical interpretation. When written in terms of the temperature, it becomes

$$[T_0^{(i)}(r)]^4 = \frac{1 - \mu_m}{2}T_w^4 + \frac{1 + \mu_m}{2}T_\infty^4. \quad (13)$$

This relation is easily derived once the gas is assumed transparent. At any point in the gas, we then have $I = \sigma T_w^4/\pi$ when $\mu_m \leqslant \mu \leqslant 1$ and $I = \sigma T_\infty^4/\pi$ when $-1 \leqslant \mu \leqslant \mu_m$. In other words, the intensity at r depends on whether the sphere or gas at infinity is being viewed. With this, equation (3) directly yields equation (12). It should be noted that equation (12) is also given by Ryhming [5].

We now introduce $Q_o = 1 = \Delta_o$ and equations (6), (10) and (11) into equation (1) and expand for small B_u. All terms of $O(1)$, $O(B_u)$, $O(B_u^2)$, and $O(B_u^2\Delta_1)$ cancel, thereby yielding

$$\tfrac{1}{2}Q_1 = -\tfrac{1}{3}\rho^3 - \rho^2 - \rho + \tfrac{1}{3}(\rho^2 + 2\rho)^{\frac{3}{2}} + \int_0^\rho(\rho_1 + 1)\theta_0^{(i)}(\rho_1)\,d\rho_1 - \int_\rho^\infty(\rho_1 + 1)\theta_0^{(i)}(\rho_1)\,d\rho_1 + \int_0^\infty(\rho_1^2 + 2\rho_1)^{\frac{1}{2}}\theta_0^{(i)}(\rho_1)\,d\rho_1,$$

where $\nu_1 = B_u$. By substituting equation (12) into the above, we find $Q_1 = -\frac{1}{3}$. Hence, Q is now given by

$$Q = 1 - \tfrac{1}{3}B_u + \ldots, \quad (14)$$

where the B_u term accounts for the reduction in heat transfer due to absorption of radiation incident on the sphere's surface.

Although the inner expansion $\Theta^{(i)}(\rho) = \tilde{\Theta}^{(i)}(\rho)/(1 + \rho)$ goes to zero as $\rho \to \infty$, it is not uniformly valid. This is because $\Theta^{(i)}$ goes to zero as $(4\rho^2)^{-1}$, whereas $\Theta^{(o)}$ goes to zero as $3B_u/4\rho$. However, when $T_w < T_\infty$, the composite solution to a first approximation is given by the leading term of the inner expansion (as a consequence of $\Theta^{(i)}$ going to zero when $\rho \to \infty$). In the $T_w < T_\infty$ case, the heat flux is $-(B_u/3) + \ldots$.

Matching and composite solution

It is easy to show [12] that the inner part of the leading term of the outer expansion is $B_u^2/4z$. It is important to note that this result stems from the E_0 term in equation (9). This means that the Rosseland diffusion result given by the $\frac{3}{4}$ term in (9) does not match the leading term of the inner expansion.

The outer part of the leading term of the inner expansion is also $B_u^2/4z$. In contrast to the large B_u case [6, 14], the leading terms match directly without determining any constants.

A composite first approximation for $\Theta(z)$ is readily shown to be [12]

$$\Theta_0^{(c)}(z) = \frac{1}{2} - \frac{1}{2}\left(1 - \frac{B_u^2}{z^2}\right)^{\frac{1}{2}} + \frac{B_u^2}{z}\left[\frac{e^{-z} - 1}{4z} + \frac{3}{4} - \left(\frac{\pi^2 - 8}{16}\right)E_2(z)\right]. \quad (15)$$

3. RESULTS AND DISCUSSION

The results for large B_u equivalent to equations (14) and (15) are [6]

$$Q = \frac{4}{3B_u} - \frac{4q(\infty)}{3B_u^2}, \qquad \Theta^{(c)} = \frac{B_u}{z} - \frac{q(z - B_u)}{z}, \qquad 1 \ll B_u \quad (16a, b)$$

where q is Hopf's function, which is tabulated in Kourganoff [15]. If we now use the leading terms from equations (14) and (16a), we obtain the interpolation formula, useful for all B_u,

$$Q = \frac{1}{1 + (3B_u/4)} \quad (17)$$

originally given by Heaslet and Baldwin [16].

Ryhming [5] and Viskanta and Crosbie [4] have given exact numerical results when there is an outer concentric sphere with a finite radius r_b. We can compare our small B_u results with theirs only when simultaneously their $\alpha a \ll 1$

and $1 \ll \alpha r_b$. The largest value of αr_b used by these authors is 10, and all results attributed to them in Figs. 1 and 2 are

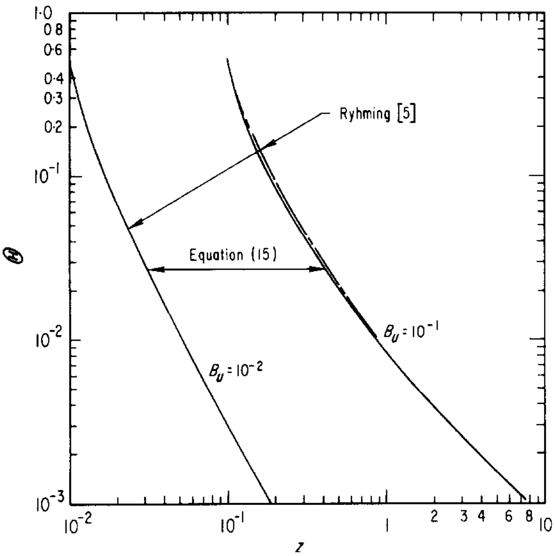

FIG. 1. $\Theta(z; B_u)$ vs z.

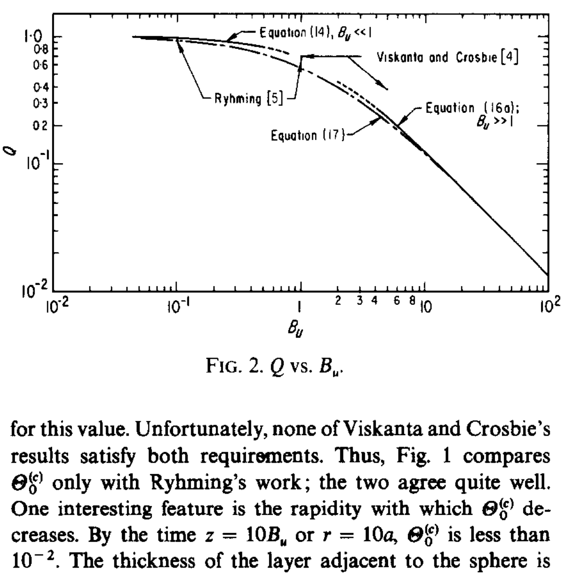

FIG. 2. Q vs. B_u.

for this value. Unfortunately, none of Viskanta and Crosbie's results satisfy both requirements. Thus, Fig. 1 compares $\Theta_0^{(c)}$ only with Ryhming's work; the two agree quite well. One interesting feature is the rapidity with which $\Theta_0^{(c)}$ decreases. By the time $z = 10B_u$ or $r = 10a$, $\Theta_0^{(c)}$ is less than 10^{-2}. The thickness of the layer adjacent to the sphere is thus of $O(a)$.

Figure 2 shows the heat transfer as given by equations (14), (16a), and (17). Only Rhyming's numerical value at $B_u = 0{\cdot}1$ agrees with the asymptotic results. His and Viskanta and Crosbie's result at $B_u = 5$ differ from equation (16a) because of the presence of the outer sphere. At $B_u = 1$, their results differ from equation (17) partly for the same reason and partly because of the ad hoc nature of equation (17).

If we evaluate relation (15) at the surface, where $z = B_u$, we obtain $\Theta_0^{(c)}(B_u) = \frac{1}{2} + O(B_u)$ in accord with Fig. 1. From this it is easy to show that T_u satisfies $T_u \geqslant (\frac{1}{2})^{\frac{1}{4}}\, T_w = 0{\cdot}841\, T_w$. There is thus a lower bound for the temperature of the gas adjacent to the wall. Although derived for a sphere, this bound is independent of B_u and therefore holds for any geometry.

An interesting approximation, initially due to Chou and Tien [2], is the method of regional averaging, sometimes referred to as a modified moment method. Hunt [3] has applied this to the concentric sphere case without a heat-generating gas. For a single sphere, his results are easily modified to yield, in the two-region approximation,

$$\Theta^{(2)} = \frac{2z - 2(z^2 - B_u^2)^{\frac{1}{2}} + 3B_u^2}{4 + 3B_u}\,\frac{1}{z}, \tag{18}$$

with $Q^{(2)}$ given by equation (17). This latter result lends further justification for equation (17). Although not shown in Fig. 1, equation (18) agrees quite well with Ryhming's results. (See Figs. 2 and 3 in [3] for a comparison of the two-region and three-region approximations with Ryhming's results.) It is easy to show that, for small B_u, equation (18) agrees with the inner expansion close to the sphere and with $3B_u^2/4z$ when z is large. (When B_u is small and z is large, the quantity $3B_u^2$ in the numerator of (18) is important. The quantity $3B_u$ in the denominator is unimportant at any z; it is important, however, when B_u is large.) Furthermore, neither $\Theta^{(2)}$ nor $Q^{(2)}$ is restricted to small B_u. Expanding equation (18) for large B_u shows that its leading term agrees with its equivalent in equation (16b). The two-region approximation therefore gives good results for Θ and Q for all z and all B_u.

The three-region approximation, which agrees closely with Ryhming's results [3], is considerably more complicated than the two-region approximation. For small B_u, one can show that $Q^{(3)} \sim 1 - \frac{11}{36}B_u + \ldots$, which agrees with equation (14).

Olfe [17] has recently put forth a modified differential approximation, which he subsequently applied to the concentric sphere problem [8]. When his results, which are more complicated than that given by the two-region approximation, are modified for a single sphere and expanded for small B_u, we obtain $Q = 1$; for the emissive power we obtain equation (18) minus the unimportant $3B_u$ term in the denominator. The quantity g, given by equation (8) in [8], must first be written in terms of the inner variable ρ before the B_u expansion is performed. The leading terms for Q and Θ are thus identical to the two-region approximation.

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TRANSIENT RADIATIVE HEAT TRANSFER IN A PLANE LAYER

K. KRISHNA PRASAD† and R. G. HERING

University of Illinois at Urbana-Champaign, Urbana, Illinois, U.S.A.

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INTRODUCTION

Energy transfer by thermal radiation within absorbing and emitting media has received considerable attention in recent years. Transient radiative transfer processes, however, have received only limited consideration. This study is concerned with unsteady energy transfer by radiation in a stationary plane layer of a non-conducting medium. Nemchinov [1] utilized a two-flux model to study transient cooling of a layer in the absence of walls while Viskanta and Bathla [2] employed an exact formulation to study the same system when the layer is symmetrically heated by an external diffuse and collimated radiant flux. The latter authors also cite a number of other transient radiative transfer studies most of which are concerned with a spherical geometry. The present study is distinguished from earlier investigations by the presence of walls and unsymmetrical boundary conditions. The system with the conditions imposed here is analogous to the conventional problem in heat conduction and, therefore, permits ready comparison with results for simultaneous conductive and radiative transfer. Exact radiative transfer methods are employed to formulate the energy transfer problem and two techniques are developed to construct solutions to the governing energy equation.

† Present address: Department of Mechanical Engineering, Indian Institute of Science, Bangalore-12, India.